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CONSUMER CHOICE WHEN THE ENVIRONMENT IS A VARIABLE: THE CASE OF RESIDENTIAL SITE SELECTION

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1. Introduction

A consumer must decide not only how to allocate his income among alternative items but also where to reside. The two decisions are of course linked. On the income side, prices of goods often depend on shipping or transport costs and also one's net disposable income depends on one's out-of-pocket commuting costs; and on the personal utility side, where one lives directly affects the amount of leisure time available after commuting time has been expended. These points have been analyzed by numerous observers.[1] Other observers have noted that one's receipt of government services as well as one's tax bill depends on where one resides.[2]

This paper redevelops the analytics of a consumer optimally selecting a site for his residence and shows that the problem of site selection is a special case of a more

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general problem of a consumer optimally selecting his environment, environment being a more general case of residential site. The point of departure is an attempt to conceptually refurbish Alonso's now classic treatment of residential site selection. It is in the reformulating of Alonso's treatment that the elements of mode selection, and supply and cost of government services are able to be incorporated into a consumer's optimal allocation of income problem.

It is of interest for the history of doctrine that the fairly general treatment of space in consumer theory in this paper is completely analogous to the treatment of space in production theory presented by Moses [7] in 1958. That analysis was conducted in terms of isoquants and firm revenue constraints with spatial dimensions.

2. Alonso's Formulation and its Problems

Alonso's model was derived from the following observation. "An individual who arrives in a city and wishes to buy some land to live upon will be faced with the double decision of how large a lot he should purchase and how close to the centre of the city he should settle". Alonso [1; p.18]. Alonso abstracted from institutional issues as well as heterogeneity in land quality and proceeded to explicitly define the consumer's optimizing problem and the problem's solution.

A consumer arrives at a city with a single central business district (CBD) and faces an anular zone of land surrounding the CBD with a land rent function that declines monotonically from the edge of the CBD to the geographic border of the city, this latter being the frontier dividing land used for residential purposes from that used for agriculture.[3] The consumer earns his income from working in the CBD and must commute at some positive cost which increases with distance travelled. The consumer spends his entire income on land (rents), commuting, and a bundle of other commodities with price index p_z. We abstract from the structure on the land; we can assume that any portion of land bought in equilibrium has an identical structure on it.[4]

Formally, Alonso's formulation of the consumer's problem is to:

maximize $U(z,q,t)$

subject to $y = p_z z + p(t)q + k(t)$

where

U	is the utility function
z	is the quantity of "other goods" desired
q	is the quantity of land
t	is distance from the CBD
y	is income
p_z	is the price of the "other goods"
$p(t)$	is the price of a unit of land at distance t from the CBD

k(t) is the cost of commuting from the CBD to a point t, the distance from the CBD.

Alonso assumes $\frac{\partial u}{\partial z} > 0$, $\frac{\partial u}{\partial q} > 0$, $\frac{\partial u}{\partial t} < 0$, $\frac{dp}{dt} < 0$, $\frac{dk}{dt} > 0$

The first order equilibrium conditions to this problem when a solution obtains in the positive orthant are:

$$\frac{U_q}{U_z} = \frac{p(t)}{p_z} ; \quad \frac{U_t}{U_z} = \frac{q\frac{dp}{dt} + \frac{dk}{dt}}{p_z} ; \quad \frac{U_t}{U_q} = \frac{q\frac{dp}{dt} + \frac{dk}{dt}}{p(t)}$$

and $y - p_z z - p(t)q - k(t) = 0$

where the U_z indicates $\frac{\partial U}{\partial z}$. The economic interpretation of the first and the last equilibrium conditions should be familiar. However the other two conditions involve the new term $q\frac{dp}{dt} + \frac{dk}{dt}$ Alonso defines this term as "the marginal cost of spatial movement" and argues that "a near location was preferred to a distant one, since commuting is generally regarded as a nuisance. This means that U_t is a marginal disutility, and has negative utility; in short that $U_t < 0$". Since $p_z > 0$ and $U_z > 0$, "it follows therefore that the expression $q\frac{dp}{dt} + \frac{dk}{dt}$ must be smaller than zero". Alonso [1; p.34] . Alonso proceeds to argue that given the negative sign on the above expression and $\frac{dk}{dt} > 0$, "We must conclude that $\frac{dp}{dt}$ is negative". Not only must $\frac{dp}{dt}$ be negative, but it must be sufficiently large in magnitude so as to make $q\frac{dp}{dt} + \frac{dk}{dt} < 0$. This latter aspect is more difficult to interpret. For example, when commuting costs vary linearly with distance, that is

$k(t)=st$ where s is the travel cost per unit distance, then $q < -\frac{s}{\frac{dp}{dt}}$. In other words, the quantity of land purchased in equilibrium must be less than the ratio of the unit distance commuting costs to the slope of the land rent function.

Observe that Alonso's analysis of the equilibrium rests crucially on the assumption that $\frac{\partial U}{\partial t} < 0$. However, the justification for the assumption is not precise. In some instances, Alonso requires $\frac{dp}{dt} < 0$ a priori. If we take this as a condition characterizing a city and not subject to change by an individual, then we find from the second order conditions characterizing an equilibrium for an individual that $q\,\frac{dp}{dt} + \frac{dk}{dt}$ need not have a negative sign, i.e. the magnitude of $\frac{dk}{dt}$ can outweigh that of $q\,\frac{dp}{dt}$.

The fact that a move from the CBD results in savings on land cost, decreases in leisure time available and increases in out-of-pocket commuting costs is not justification for assuming $U_t < 0$.

The second order equilibrium conditions indicating that a maximum rather than a minimum has been obtained are

i) $U_{zz} < 0$

ii) $U_{zz}\, U_{qq} > 0$

iii) $U_{zz}\, \{U_{qq}(U_{tt} - \lambda q\,\frac{d^2p}{dt^2} - \lambda\frac{d^2k}{dt^2}) - (\lambda\frac{dp}{dt})^2\} < 0$

or $(U_{tt} - \lambda q\frac{d^2p}{dt^2} - \lambda\frac{d^2k}{dt^2}) < 0$

and $U_{zz}U_{qq}\left(U_{tt} - \lambda q \frac{d^2p}{dt^2} - \lambda \frac{d^2k}{dt^2}\right) > U_{zz}\left(\lambda \frac{dp}{dt}\right)^2$

iv) $$U_{zz}\{U_{qq}(-(q \frac{dp}{dt}+\frac{dk}{dt})^2)+\lambda\frac{dp}{dt}(-(p(t)-(q\frac{dp}{dt}+\frac{dk}{dt})))$$

$$-p(t)\left[\lambda \frac{dp}{dt}\left(q\frac{dp}{dt}+\frac{dk}{dt}\right)-p(t)\left(U_{tt}-\lambda q\frac{d^2p}{dt^2}-\lambda\frac{d^2k}{dt^2}\right)\right]\}$$

$$p_z\{-p_z\left[U_{qq}\left(U_{tt}-\lambda q\frac{d^2p}{dt^2}-\lambda\frac{d^2k}{dt^2}\right)-\left(\lambda\frac{dp}{dt}\right)^2\right]\}>0$$

where for example $U_{zz} \equiv \frac{\partial^2 U}{\partial z^2}$ and λ is the Lagrangian multiplier or shadow price on income.

Conditions (i) and (ii) are orthodox results indicating diminishing marginal utilities with respect to "other goods" and land. In condition (iii) when commuting costs vary linearly with distance travelled, $\frac{d^2k}{dt^2} = 0$ and we get $q > \frac{U_{tt}}{\lambda \frac{d^2p}{dt^2}}$. This inequality implies a particular quantitative restriction on the quantity of land purchased in equilibrium. We cannot say a priori what sign U_{tt} will have or what the sign of $\frac{d^2p}{dt^2}$ should be. Condition (iv) is too complicated to interpret.

The difficulty in interpreting the equilibrium conditions in Alonso's model suggests that the specification or precise formulation of the model may be unsatisfactory. Alonso was attempting to determine the equilibrium solution to the problem of an individual choosing his location of residence in a city "by means of classical consumer equilibrium theory". The beauty of classical consumer theory lies in part in the simplicity of its mathematical form and the ease with which one can interpret the equilibrium conditions. Alonso has lost many of these qualities in his formulation of the problem of an individual choosing his location of residence in a city.

3. A Revised Model of Residential Site Selection

Alonso's fundamental insight was the recognition of the qualitative effects of residential site choice on a consumer's utility and income. However the introduction of distance as an argument in the utility function led to the peculiar quality of his model judged in terms of the nature of its equilibrium conditions. Distance is really a proxy variable for leisure. One has disutility for distance traversed because it represents leisure time foregone.

We can reformulate the residential site choice model by integrating the trade off between income and leisure into the consumer's utility maximization problem[5]. We are able to overcome the idiosyncratic character of Alonso's formal model and integrate the income-leisure choice into a more general model of consumer choice.

The consumer is assumed to face an economic environment the same as Alonso's. His utility is defined to depend on the amount of land he consumes (this is a proxy for the quality of his residence), the amount of a composite of other commodities he consumes, and the amount of leisure he has available to him. His income constraint is the same as the one Alonso presented.

The amount of leisure available is assumed to depend inversely on the distance the individual resides from the CBD. We abstract from other factors which affect the amount of leisure available for now, since the other factors are assumed to be quantitatively much less significant[6]. The consumer's choice can for expository convenience be separated into a two stage process. A consumer chooses a site, $\bar{t}$ distance from the CBD. The amount of leisure he will have available $L(\bar{t})$ becomes fixed as well as his out-of-pocket commuting expenses $k(\bar{t})$. The price of land will also become fixed at $p(\bar{t})$ also. The consumer then performs a classical optimization calculation.

$$\text{maximize} \quad U(z,q,L(\bar{t})) \tag{1}$$

$$\text{subject to} \quad y-k(\bar{t})=p_z z+p(\bar{t})q \tag{2}$$

where U is assumed to be quasi-concave. The problem is a simple text book problem in consumer theory with two variables. Recall that L, k, and p(t) are fixed once t is specified. There will be an optimal bundle $(z,q;\bar{t})$ chosen at point $\bar{t}$. The equilibrium condition is the familiar one.

$$\frac{U_z}{U_q} = \frac{p_z}{p(\bar{t})}$$

The consumer then selects an alternative site with a different distance $\bar{\bar{t}}$ and repeats the optimization in (1) and (2). The individual will be able to rank his relative utilities at $\bar{t}$ and $\bar{\bar{t}}$ and choose the better site. The complete choice problem is for the consumer to test all feasible distances and to select the one where he is best off.

Figure 1 illustrates the consumer's equilibrium position given alternative distances, or t's.

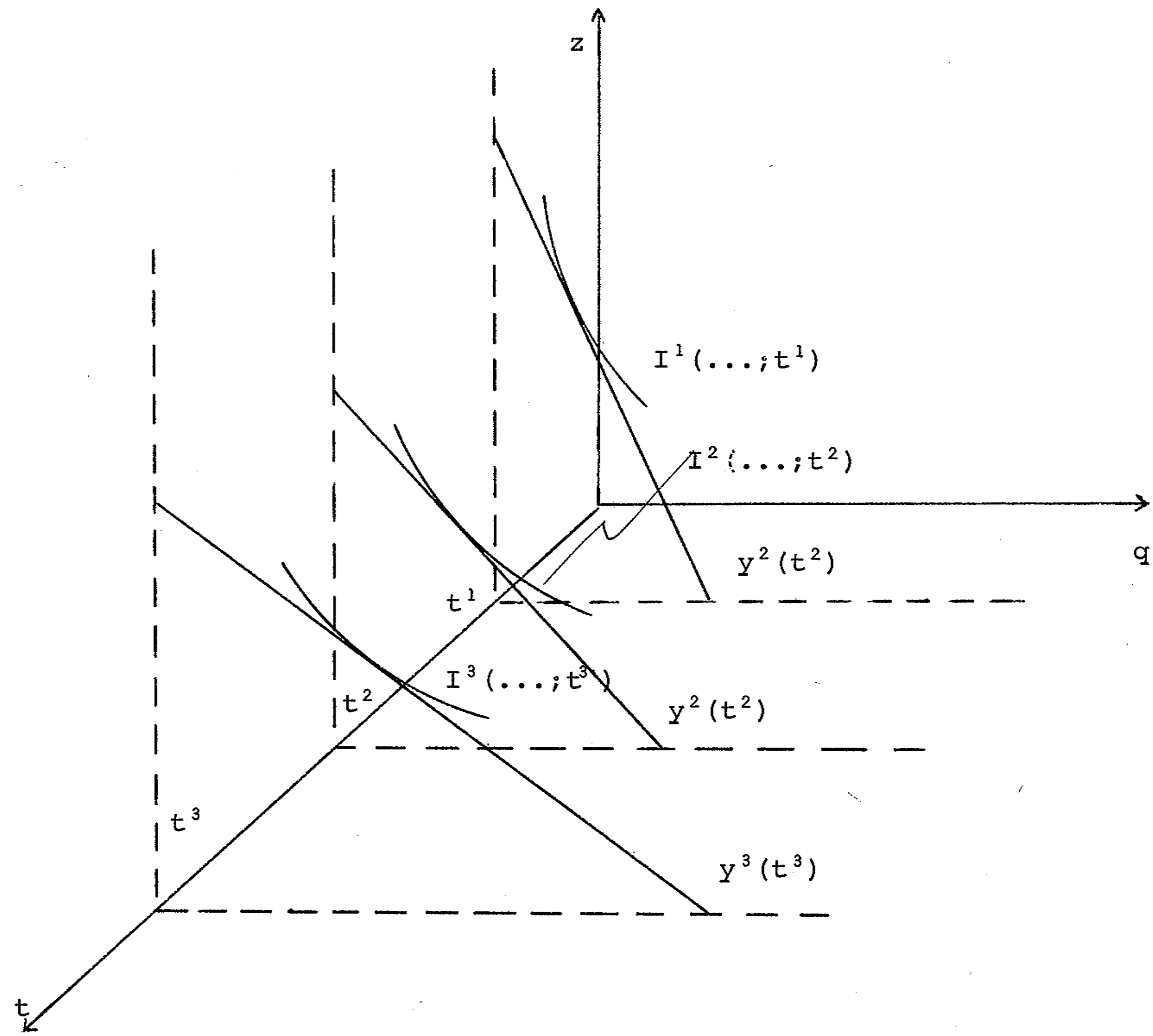

Figure 1

In Figure 1, at a site with distance t^2 from the CBD, a consumer faces a site specific set of relative prices for "other commodities" and land $p_{z/p(t^2)}$ and an implied income constraint y^2. A site specific amount of leisure will be available and an indifference curve I^2, tangent to the budget constraint will be mapped out corresponding to a maximum utility index U^2.

In Figure 2 is another diagram which can be derived from Figure 1.

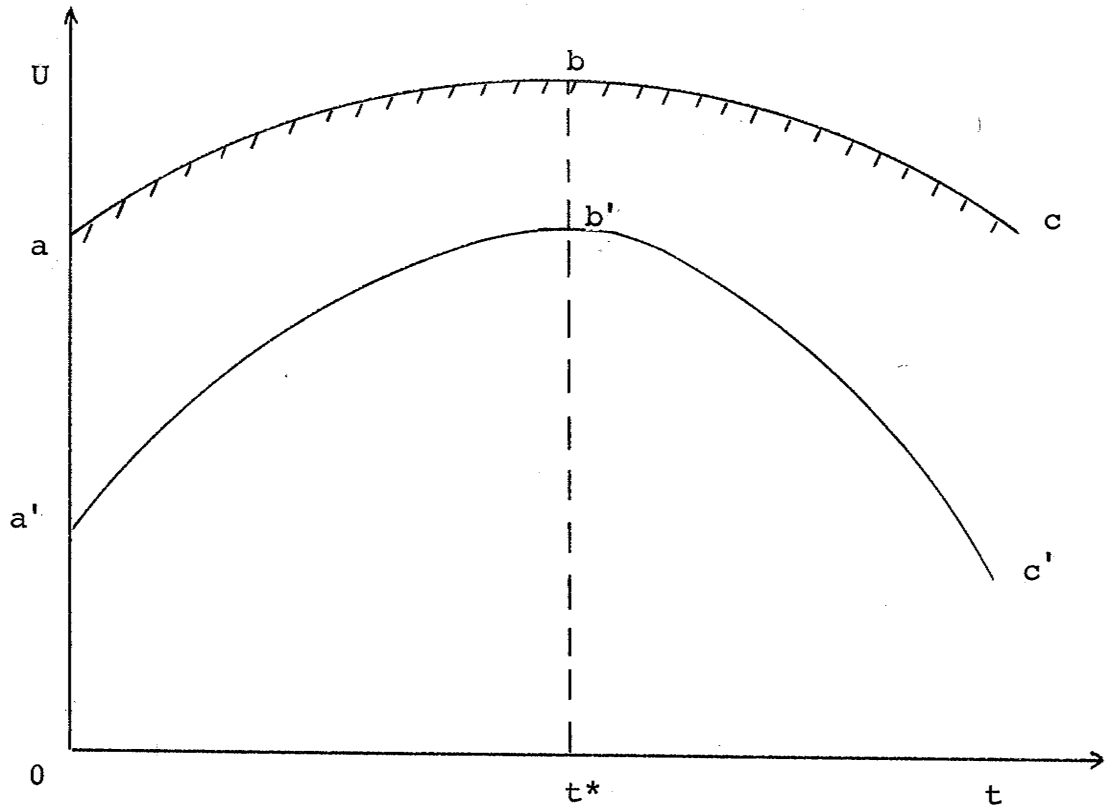

Figure 2

In Figure 2, we have schedule abc tracing out the frontier of maximum attainable utility for any feasible site or distance t. At the site corresponding to t* the individual can attain his maximum utility. Observe that schedule abc corresponds to an ordinal indicator, U. Any alternative indicator which ranks sites in the same way would suffice. For example schedule a' b' c' corresponds to a new utility indicator which is a monotone transformation of U.

The shape of schedule abc will differ for different individual utility functions. In particular, it need not be concave or monotonic. Some individuals will generally choose to locate on the edge of the CBD and some others on the edge of the city.

Formally the consumer's choice problem is to sumultaneously locate himself and allocate his income to goods and services desired. We have

$$\text{maximize} \quad U(z,q,L(t);t)$$

$$\text{subject to} \quad y = p_z z + p(t)q + k(t)$$

where U is quasi-concave. We assume that a consumer desires more of a good than less and hence that $\frac{\partial U}{\partial z}, \frac{\partial U}{\partial q}, \frac{\partial U}{\partial L}$ are all positive.

The first order equilibrium conditions are

$$\frac{U_z}{U_q} = \frac{p_z}{p(t)} \; ; \; \frac{U_L}{U_z} = \frac{q\,\frac{dp}{dt} + \frac{dk}{dt}}{\frac{\frac{dL}{dt}}{p_z}} \; ; \; \frac{U_L}{U_q} = \frac{q\,\frac{dp}{dt} + \frac{dk}{dt}}{\frac{\frac{dL}{dt}}{p(t)}}$$

and $\quad y-p_z z-p(t)q-k(t) = 0$

Given the assumptions on signs, we must have the price of a unit of leisure

$$\frac{q\,\frac{dp}{dt} + \frac{dk}{dt}}{\frac{dL}{dt}} > 0$$

and since $\frac{dL}{dt} < 0$, then $q\frac{dp}{dt} + \frac{dk}{dt} < 0$. This latter result is identical with Alonso's, except that we have derived it by a different route in a different model.

4. Alternative Transportation Modes

We have assumed in Section 3 that commuting costs, both in money terms and in terms of leisure foregone, could be defined simply in terms of distance or location. We abstracted from the possibility of the consumer substituting alternative modes. Clearly the type of mode selected will affect the amount of leisure available and will also affect the amount of income available for other purposes.

If we now specify the location t and mode m, the consumer will have his relative prices fixed, leisure fixed, and income fixed. He will then have a familiar optimization problem facing him given t and m. That is he will

$$\text{maximize} \quad U(z,q,L(\bar{t},\bar{m});\ \bar{t},\bar{m})$$

$$\text{subject to} \quad y-k(\bar{t},\bar{m}) = p_z z+p(\bar{t})q$$

where L(t,m) is the amount of leisure available as a function of distance from the CBD and mode of transport, and k(t,m) is the money cost of transport as a function of distance t and mode m.

For any given mode and distance, in equilibrium the consumer will equate the marginal rates of substitution between land and "other goods" to the ratio of the prices of land and "other goods".

$$\frac{U_q}{U_z} = \frac{p(\bar{t})}{p_z}$$

These equilibrium conditions define a subset of admissable optima and the optimum optimorum will occur where the pair t and m are selected so that the consumer attains the highest possible indifference curve. Figure 3 is analogous to Figure 2. In Figure 3 the environment is defined by two variables,

t and m, as opposed to one variable t, used in Figure 2.

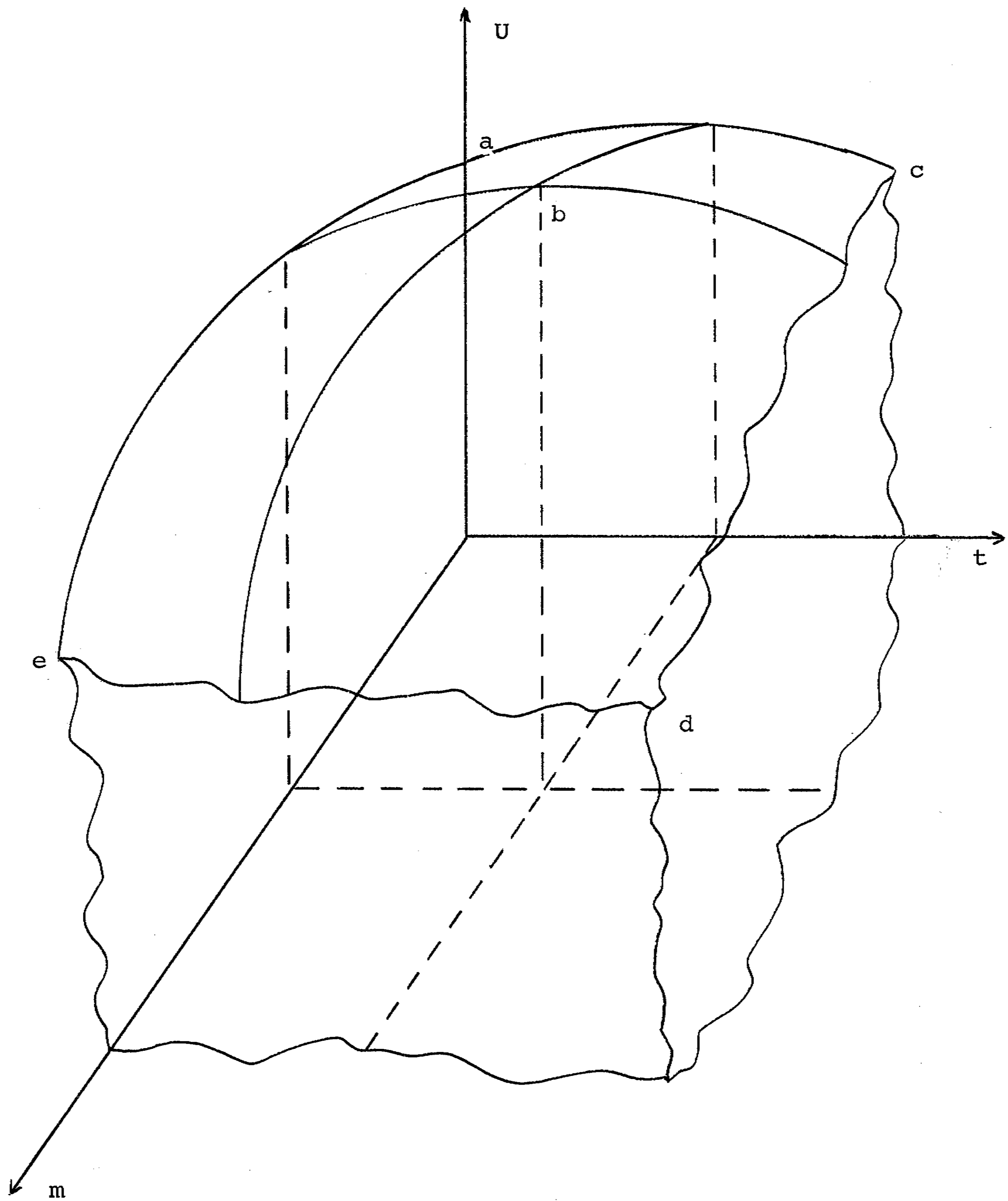

Figure 3

The consumer will vary his environment by altering his location and transportation mode to alter his quantity of income and leisure received so as to obtain his utility optimum at b in Figure 3. Note all points on surface acde define situations where the consumer has spent all his income and has equated his marginal rate of substitution between land and "other goods" to ratio of prices for land and "other goods" respectively. Note again that surface acde need not be concave throughout with an interior maximum point such as the point b in Figure 3.

5. The Consumption of Public Goods

That the quantity and quality of public goods one receives as well as one's tax bill varies from location to location is now widely recognized. A consumer's environment in a general sense determines for an individual the amount of public goods he receives and his tax bill. Public goods provided by the government are analogous to leisure for an individual consumer and taxes are analogous to out-of-pocket commuting expenses.

We can set down formally the consumer's solution to his problem of choice. Let us revert to defining the environment in terms of the single dimension, location, or distance t from the CBD. Our consumer now tests a site or chooses a distance $\bar{t}$. Given $\bar{t}$, the price of land $p(\bar{t})$ is fixed for the consumer, the commuting costs $k(\bar{t})$, the leisure available $L(\bar{t})$, the tax bill $T(\bar{t})$, and the volume of public goods received $G(\bar{t})$ are fixed. For specific site $\bar{t}$, the consumer's choice problem is the solution of

$$\text{maximize} \quad U(z,q,L(\bar{t}),\ G(\bar{t});\ \bar{t})$$

$$\text{subject to} \quad y-k(\bar{t})-T(\bar{t}) = p_z z + p(\bar{t})q$$

The first order equilibrium conditions to this problem are that all income is expended and

$$\frac{U_z}{U_q} = \frac{p_z}{p(\bar{t})} \tag{3}$$

The consumer's complete problem is to select a site t such that the first order equilibrium conditions are satisfied and the highest level of satisfaction is received at the t chosen compared with all alternative feasible t's.

We find in the above problem that the environment or site in this case affects the consumer's optimal position through additional variables in both the utility function and in the income constraint. Moreover, the implications of the above model for the analysis of residential choice are noteworthy. A consumer does not simply assess his personal cost-benefit position with respect to his receipt of public goods and disbursements for taxes and plan his choice of residence site.

He considers his overall relative welfare position at alternative sites in a framework in which taxes and benefits are constituents of a much more general calculation. For example it is no less valid (or invalid) for a consumer to weigh his out-of-pocket commuting expenses rather than his taxes against his receipt of public goods. Either of the two assessments is incorrect since the consumer's relative welfare calculation is a general one in which taxes and quantities of public goods enter in a simultaneous fashion.

6. Consumer Choice when the Environment is a Variable

Location and mode of travel are two parameters we have examined which affect a consumer's relative welfare and yet do not directly afford utility to the consumer. Location and travel mode are the two dimensions which defined the environment for our consumer. Given his desired environment, our archetypal consumer then expended his income so as to maximize his utility in the familiar textbook fashion.

Environmental factors affected the consumer's relative welfare calculation through his income constraint and through his utility function.

We can extend consumer theory by incorporating general environments into the individual utility optimization paradigm. Let us define an environment in terms m dimensions $(t_1,\ldots,t_m)$. A dimension of the consumer's environment t_i will be a variable which the consumer can affect in a predictable way either by moving (e.g. location) or by expending alternative amounts of income (e.g. mode of travel) and the variable does not represent an entity which yields utility to the consumer directly. An environmental variable t_i differs from a conventional good or service x_i in the respect that x_i is desired in its own right whereas t_i is desired for its capacity to affect relative individual welfare indirectly. Location would be an environmental variable whereas apples would be conventional goods.

The environmental dimensions determine, for a chosen environment, the quantities of a set of goods or services consumed by the individual but not traded in conventional goods and services markets. Typically this set of goods $x_j(t_1,...,t_m)$; $j= k+1,...,n$ will be dominated by items normally labelled "externalities". There will be private externalities such as neighbourhood noise, ugliness (or attractiveness) and public externalities such as governmentally provided goods and services such as police protection.

There will typically be a set of prices facing the consumer which are environmentally dependent. Any price facing the consumer which varies with the consumer's location is an example. This set of environmentally dependent prices will be $p_j(t_1,...t_m)$; $j = h+1,...,k$. Finally there will be a set of disbursements from income which are fixed when the environment is chosen. This set will be $C_j(t_1,...,t_m)$; $j = 1,...,r$. Items such as municipal taxes are typical examples although we observed out-of-pocket commuting costs to be another important one.

The consumer's general optimizing problem when his environment is a variable is to select $x_1,\ldots,x_k$, and $t_1,\ldots,t_m$ so as to maximize:

$$U(x_1,\ldots,x_k,x_{k+1}(t_1,\ldots,t_m),\ldots,x_n(t_1,\ldots,t_m);t_1,\ldots,t_m)$$

subject to

$$y = p_1x_1+\ldots+p_hx_h+p_{h+1}(t_1,\ldots,t_m)x_{h+1}+\ldots$$

$$+p_k(t_1,\ldots,t_m)x_k+C_1(t_1,\ldots,t_m)+\ldots+C_r(t_1,\ldots,t_m)$$

$$x_1,\ldots,x_n \geqq 0$$

The first order equilibrium conditions characterizing a consumer's optimum position are

i) $\frac{U_i}{U_j} = \frac{p_i}{p_j}$ $(i,j, = 1,\ldots,k)$

ii) $y - \sum_{i=1}^{k} p_ix_i - \sum_{j=1}^{r} C_j = 0$

$$\text{iii)} \quad \frac{U_i}{\sum\limits_{j=k+1}^{n} U_j \dfrac{\partial x_j}{\partial t_l}} = \frac{p_i}{\sum\limits_{s=h+1}^{k} x_s \dfrac{\partial p_s}{\partial t_l} + \sum\limits_{v=1}^{r} \dfrac{\partial C_v}{\partial t_l}} \qquad \begin{array}{l} i=1,\ldots,k \\ l=1,\ldots,m \end{array}$$

Conditions (i) and (ii) are familiar results appearing in contemporary textbooks in price theory. Condition (iii) is a new result indicating that the consumer's optimal environment has been reached. Note that conditions (i) - (iii) simultaneously define an optimum optimorum for the individual; the choice of an optimum environment cannot be separated from the orthodox theory of consumer behaviour.

Buchanan [3] has in his theory of clubs developed a model of consumer choice when the environment is a variable. He defines his environment in terms of the number of individuals jointly consuming a good or service. The set of consumers jointly consuming forms a "club". In traditional theory, "Implicitly, the size for sharing arrangements is assumed to be determined exogenously to individual choices. Club size is presumed to be a part of the environment", Buchanan [3; p.5]. Buchanan makes club size a dimension which the consumer selects. Club size can readily be considered an element of the general environment defined above. Since one's taste for club size is actually a surrogate measure for one's evaluation of the positive or negative externalities generated from fellow consumers, it seems more satisfactory to introduce club size in the general environment vector. The affects of alternate club sizes would be registered through Pigovian externalities in the utility function and through income changes via possible cost sharing agreements.

7. Concluding Remarks

By means of distinguishing between variables such as goods and services and those defining an environment facing an individual, we have been able to reformulate Alonso's model of residential site choice along lines typical of classical consumer theory. The reformulation has permitted us to incorporate the affects of alternate travel modes, and the costs and availability of public goods on an individual's residential site selection. Alternatively given that the economic landscape of many cities is continually changing, we can consider the revised Alonso model as a prelude to a model of intra-urban migration. The analytics of job selection have been omitted so we do not have a general model defining the comparative statics of migration.

In Section 6, we observed that the model of residential site selection is a special case of a more general theory of consumer choice. The usefulness of this more general approach will depend on the possibility of conceptually separating a property of the environment from a convential good or service. With location the distinction seems clear; with mode of travel the distinction is slightly less so. The problems of distinguishing qualities in other cases are not of an order of magnitude different from those in the utility tree approach to consumer theory or in the kindred Lancaster - "new approach" to consumer theory.

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FOOTNOTES

1. See for example Wingo [12], Alonso [1], Becker [2], Harris, Tolley and Harrell [4], for treatments of the choice problem facing a single individual. Muth [8], [9], Mills [6], Pines [10], and Pines and Hockman [5] have dealt with individual choices aggregated to market demand and supply relations.

2. Tiebout [11] presented the pioneering treatment of this subject.

3. Throughout this paper, place of employment could be substituted for CBD. The use of CBD implies a city with a single peaked rent-distance cone but cities with multiply peaked rent-distance surfaces are possible.

4. If we consider a residential structure as a mechano set with a single price per unit of structure, then housing will simply be lumped in with other commodities or goods.

5. Other observers have treated the leisure variable explicitly in the utility function e.g. Harris, Tolley and Harrell [4] but have not developed the analysis along the lines of this paper.

6. The analysis in this paper no where incorporates the classical treatment of the labour-leisure choice through variations in hours spent at income earning.

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